

Cambridge International AS & A Level

CANDIDATE
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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1** The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point $(4, 5)$.

Find the equation of the curve.

[3]

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- 2** The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q . It is given that $p + q = 276$.

Find the possible values of the constant a .

[4]

[illegible]

- 3 (a) Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p . [2]

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- (b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

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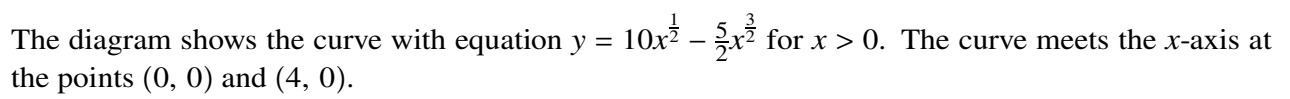
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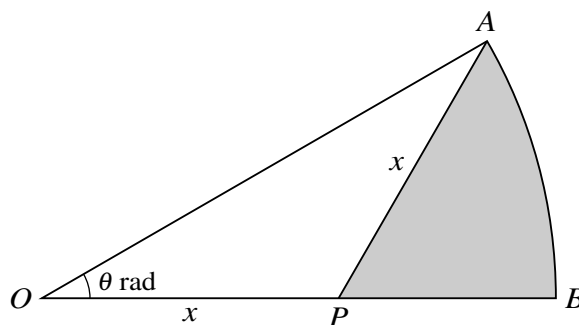
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4 Solve the equation $8x^6 + 215x^3 - 27 = 0$.

[3]

[illegible]

[illegible]



(a) Show that the arc length AB is $2x\theta \cos \theta$. [2]

[illegible]

(b) Find the area of the shaded region APB in terms of x and θ . [4]

[illegible]

- 7 (a) (i) By first expanding $(\cos \theta + \sin \theta)^2$, find the three solutions of the equation

$$(\cos \theta + \sin \theta)^2 = 1$$

for $0 \leq \theta \leq \pi$.

[3]

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- (ii) Hence verify that the only solutions of the equation $\cos \theta + \sin \theta = 1$ for $0 \leq \theta \leq \pi$ are 0 and $\frac{1}{2}\pi$. [2]

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- (b) Prove the identity $\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} \equiv \frac{\cos \theta + \sin \theta - 1}{1 - 2 \sin^2 \theta}$. [3]

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- (c) Using the results of (a)(ii) and (b), solve the equation

$$\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} = 2(\cos \theta + \sin \theta - 1)$$

for $0 \leq \theta \leq \pi$.

[3]

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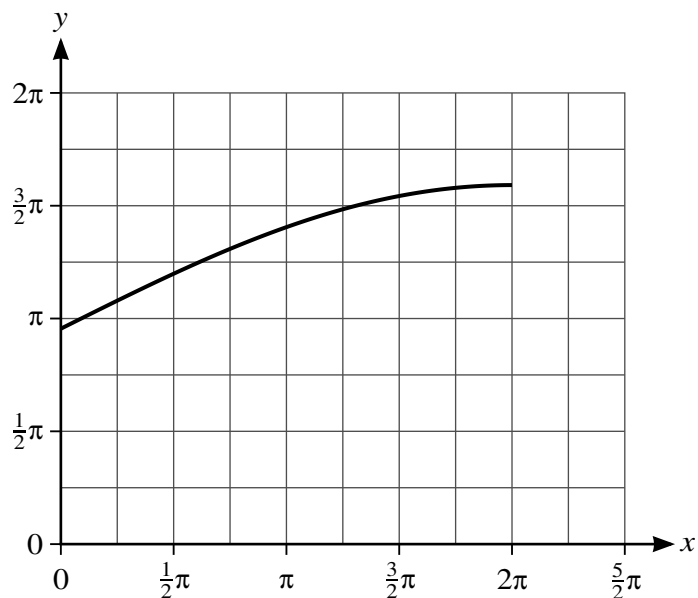
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The diagram shows the graph of $y = f(x)$ where the function f is defined by

$$f(x) = 3 + 2 \sin \frac{1}{4}x \text{ for } 0 \leq x \leq 2\pi.$$

(a) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]

(b) Find an expression for $f^{-1}(x)$. [2]

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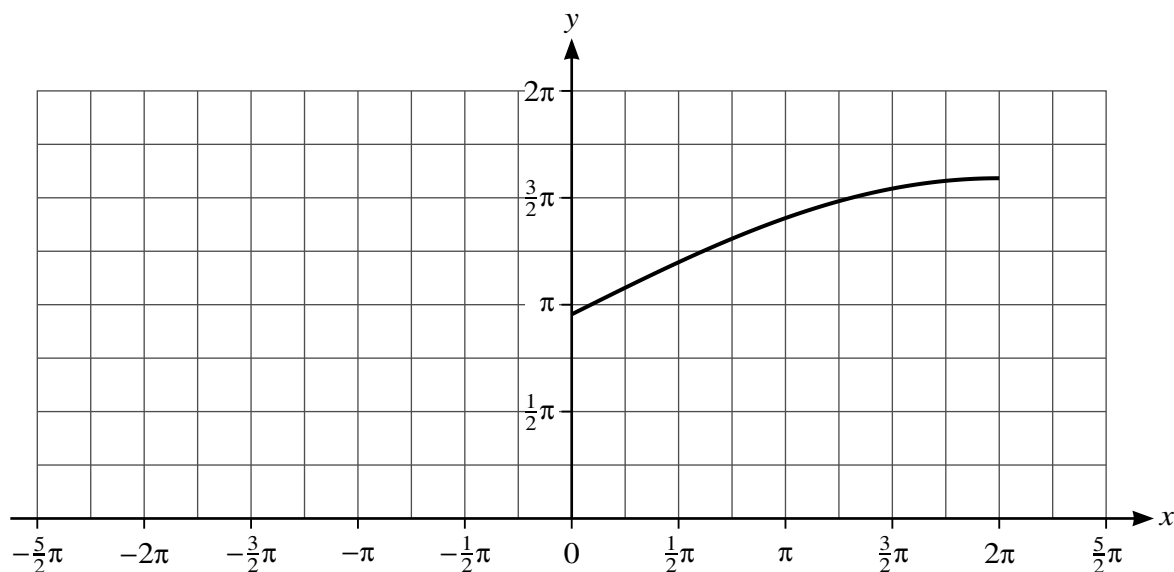
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(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2 \sin \frac{1}{4}x$ for $-2\pi \leq x \leq 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

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(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leq x \leq \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

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- 9** The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term.

[4]

[illegible]

- (b)** Show that the n th term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the n th term of the other geometric progression. [4]

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- 10** The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P .

- (a) Show that one possible value of a is 4 and find the other possible value. [5]

[illegible]

- (b) For $a = 4$, find the equation of the normal to the circle at P . [4]

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- (c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in (b). [4]

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- 11** The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

- (a)** Find $\frac{dy}{dx}$. [2]

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- (b)** Find the x -coordinate of the stationary point in terms of k . [2]

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- (c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x -axis. [4]

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[illegible]

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